PART ONE 30 POINTS

1) (10 points) Let X, and Y be two discrete random variables, their joint probability distribution is given by $f(x,y) = \frac{x-y}{18}$ for x= 1,2,3 and y= -1,0,1

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a) Find the marginal probability distribution functions of X and of Y (4 points)

$$f_1(x) = \frac{x+1+x+0+x-1}{18} = \frac{3x}{18} = \frac{x}{6} \quad x=1,2,3$$

$$f_2(y) = \frac{1-y+2-y+3-y}{18} = \frac{6-3y}{18} \quad y=-1,0,1$$

b) Calculate E(X+Y) (4 points)

$$\sum (x+y) f(X,Y) = 0 + 1/18 + 0 + 3/18 + 4/18 + 3/18 + 8/18 + 9/18 + 8/18 = \frac{36}{18} = \boxed{2}$$

E(x)

x=1: 1-1=0, 1+0=1, 1+1=2, 2-1=1, 2+0=2, 2+1=3 | 3-1=2, 3+0=3, 3+1=4

f(x)

$\frac{1+1}{18}$ = 2/18, 1 = 1/18, 1 = 0/18, 2+1 = 3/18, 2+0 = 2/18, 2-1 = 1/18 | 3+1 = 4/18, 3+0 = 3/18, 3-1 = 2/18

c) Are X and Y independent? Give reasons for your answer (2 points)

They are dependent

Why?

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2) (10 points) Let X, and Y be two continuous random variables , their joint probability density function given by $f(x,y) = \begin{cases} \frac{1}{10}(x+y) & for\ 0 \le x \le 1\ and\ 0 \le y \le 4 \\ 0 & otherwise \end{cases}$

a) Find P(0 < Y < 2 | X=0) (6 points)

$P(0<Y<2 \mid X=0)$

$1.\ \frac{g(0<Y<2,\ X=0)}{\cancel{g_2(x)}} = \frac{f(0<Y<2,\ X=0)}{f_1(X=0)}$

$f_1(x) = \frac{1}{10}\int_0^4 (x+y)\,dy$

$= \frac{1}{10}\left. xy + \frac{y^2}{2}\right]_0^4$

$= \frac{1}{10}\left[4x+8\right]$

$p = \int_0^2 (1)\,dy$

$= \frac{y/10}{8/10} = \cancel{y/8}$

$P = \int_0^2 g(0<Y<2,\ X=0)\,dy = \left.\frac{y^2}{16}\right]_0^2$

$= \boxed{\frac{1}{4}}$

b) Calculate Cov(X,Y), knowing that $E(X) = \frac{8}{15}$ and $E(Y) = \frac{38}{15}$ (4 points)

$E(X,Y) = \frac{1}{10}\iint (xy)(x+y)\,dx\,dy$

$= \frac{1}{10}\int_0^4\int_0^1 \left(x^2y + xy^2\right)dx\,dy$

$\frac{1}{10}\int_0^4 \left[\frac{x^3}{3}y + \frac{x^2}{2}y^2\right]_0^1 dy$

$= \frac{1}{10}\int_0^4 \left[\frac{1}{3}y + \frac{1}{2}y^2\right] dy$

$= \frac{1}{10}\left[\frac{1}{3\times2}y^2 + \frac{1}{2\times3}y^3\right]_0^4$

$=$

$= \boxed{\frac{40}{30}} = \boxed{\frac{20}{15}} \quad \frac{8}{3} + \frac{32}{3}$

$cov(X,Y)$

$= E(X,Y) - E(X)\times E(Y)$

$= \frac{20}{15} - \frac{38\times8}{15\times15}$

$= -0.0178$

3) (10 points) The resistances of certain type of resistors are independent, normally distributed, with a mean of 150 ohms, and a standard deviation of 9 ohms.

a) If one resistor is randomly selected, what is the probability that it has a resistance between 146 and 154 ohms? (3 points)

$\mu = 150 \quad \sigma = 9$

$P(146 < X < 154)$

$P\left(\frac{146-150}{9} < X < \frac{154-150}{9}\right)$

$P(-0.444 < Z < 0.444) = P = 0.888$

$= 88.8\%$

b) Suppose that 100 of these resistors were randomly selected. Let X a random variable representing the number of resistors among 100, those with resistance between 146 and 154. Find P(25 ≤ X ≤ 50) (7 points)

$\mu = np = 100 \times 0.888 = 88.8$

$q = 100 - 88.8 = 11.2$

$\sigma = \sqrt{88.8 \times q} = \sqrt{88.8 \times 0.112} = 3.15$

$P(25$

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PART TWO 70 POINTS

For each of the following questions, circle the right answer. There is only one correct answer for each question. If you circle more than one answer per question, your answer would be considered incorrect

***To construct a confidence interval for the population variance σ^2 we use a χ^2 distribution, but only when

a) The sample size n is large
b) The sample size n is large, and the population is normal
c) The population is normal
d) None of the above

***The NDU registrar Mrs. L. Eid claims that the mean GPA of NDU students is 2.55. To check the claim, the null and alternative hypotheses will be written as:

a) $H_0: \mu \neq 2.55$ against $H_1: \mu = 2.55$
b) $H_0: \mu = 2.55$ against $H_1: \mu > 2.55$
c) $H_0: \mu = 2.55$ against $H_1: \mu < 2.55$
d) $H_0: \mu = 2.55$ against $H_1: \mu \neq 2.55$

***If a random sample of size 11 selected from a normal population shows $\bar{x} = 20$, and $S^2 = 20$, then a 95% confidence interval for the population standard deviation σ will be:

a) [3.125 , 7.848]
b) [9.764 , 61.595]
c) [17.045 , 22.955]
d) None of these

$n = 11 \quad \bar{x} = 20 \quad s^2 = 20$

$1 - 0.95 = 0.05$

$\chi^2_{(0.025)(10)} = 20.483$

$\sqrt{\dfrac{10 \times 20}{20.483}} = \sqrt{9.765} = 3.125$

$\chi^2_{(1-0.025)(10)} = 3.247$

$\sqrt{\dfrac{10 \times 20}{[illegible]}}$

*** A company produces certain type of resistors, but because of some technical problems, some of the produced resistors do not meet the required specifications. We want to construct a 90% confidence interval for P " the proportion of the resistors that fail to meet the specifications" How large the sample size n must be in order to have the length of the resulting interval be 0.1?

a) 271 b) 68 c) 270 d) None of these

$1 - 0.90 = 0.1$

$\frac{\alpha}{2} = 0.05$

$z_{0.05} = 1.645$

$n = \left[\dfrac{z_{\alpha/2}}{2\beta}\right]^2 = 6.765 = 68$

$= 9.706$

***In a certain mall, 60% of customers pay cash money, 30% pay by credit card, and 10% pay by a check. Five randomly selected customers are ready to pay their bills, find the probability that 3 will pay cash money, and 2 will pay by credit card.

a) 0.022 b) 0.019 c) 0.1944 d) None of these

$n=5 \quad \frac{5!}{3!\times 2!\times 0!} \quad 0.6^3 \times 0.3^2 \times 0.1^0 = 0.1944$

*** In a certain mall the service time/ customer is a random variable X with a mean of 4 minutes and a variance of 1.96 . Find the probability that the total service time to serve 50 customers is less than 3 hours

X B

a) 0.4783 b) 0.0217 c) 0.9783 d) 0.5217

$\mu = 4m \quad \sigma = 1.96 \quad n = 50$

$P(X < 3h) = P(X < \frac{180}{50}) = \cancel{P(Z < \frac{180-4}{\frac{1.96}{\sqrt{50}}})}$

$= P(X < 3.6)$

$P(Z < \frac{3.6-4}{\frac{1.96}{\sqrt{50}}} = \cancel{-1.44} = -2.02$

$Z = 0.4783 \quad P = 0.52$

*** If the following table shows the joint probability distribution of the two random variables X, and Y

then P(X=1 | Y= 1) equals to

$\frac{2/15}{6/15} = 2/6$

X \ Y	1	2
0	$\frac{1}{15}$	$\frac{2}{15}$
1	$\frac{2}{15}$	$\frac{3}{15}$
2	$\frac{3}{15}$	$\frac{4}{15}$

a) $\frac{2}{5}$ b) 1 c) $\frac{2}{6}$ d) None of these

***If a random sample of size 16 selected from a normal population shows that $\bar{x} = 15$, And S=3, then a 95% confidence interval for the population mean μ is:

A

a) [13.40 , 16.60]
b) [13.68 , 16.32]
b) [13.53 , 16.47]
d) None of these

$\bar{x} = 15 \quad n = 16 \quad S = 3$

$\mu - z_{\alpha/2} \frac{S}{\sqrt{n}}$

$1-.95 . \quad z_{0.025} = 1.96$

*** X and Y are two continuous random variables. If their joint probability density function is given by $f(x,y) = \begin{cases} Kx(2-y) & for\ 0 \le x \le 1\ and\ 0 \le y \le 2 \\ 0 & otherwise \end{cases}$, then K equals to

a) 4 b) $\frac{1}{2}$ c) 2 d) None of these

$2xy - \frac{y^2}{2} \Big]_0^2$

$4x - 2$

$\frac{4x^2}{2} - 2x$ $2x^2 - 2x \Big]_0^1$

$\iint$

$K \int_0^2 \int_0^1 (2x - y)\, dx\, dy = 1$

$\frac{2x^2}{2} - yx \Big]_0^1$ $\int_0^2 1 - y = y - \frac{y^2}{2} \Big]_1^2 = 2 - 2 - 1 + \frac{1}{2} = -\frac{1}{2}$

$-\frac{1}{2}K = 1$ $K = 2$

$\frac{2x^2}{2} - yx \Big]_0^1$ $1 - y$ $y - \frac{y^2}{2} \Big]_0^2 = 2 - 2 = 0$

*** We have a normal population with unknown mean μ and a known standard deviation σ = 11, from which we are selecting a random sample of size n, and calculating $\bar{x}$, then we set μ= $\bar{x}$ How large n must be, if we want to be 95% confident that the error of estimate is maximum 2?

a) 116 b) 82 c) 117 d) None of these

$\sigma = 11$ $1 - 0.95 =$ $z_{0.025} = 1.96$

$n = \left[\frac{1.96 \times 11}{2}\right]^2 = 116.2$

***In constructing a 95% confidence interval for μ using the data obtained from a large sample we got the interval [85.1 , 94.9]. If the same sample data were used to construct a 99% confidence interval, then we shall get

a) [84.18 , 95.82]
b) [83.56 , 96.44]
c) None of the above
d) No enough data to construct the required interval

X B

95% $1 - 0.95 =$ $z_{0.025} = 1.96$

$\mu - \frac{1.96 \times S}{\sqrt{n}}$

*** If a random sample of 60 NDU students with GPA above 3, shows that 33 of them are males, then a 98% confidence interval for the proportion P of female NDU students with GPA above 3 is

$n = 60$ $\quad p = \frac{27}{60} = 0.45$

a) [0.3 , 0.6]
b) [0.4 , 0.7]
c) [0.28 , 0.62]
d) None of these

X A

*** X and Y are two random variables with E(X) = 2, E(Y) = 5, Var(X) = 1 , Var(Y) = 2 , and Cov(X , Y) = 0.5. If W = 2X – 4Y , then E(W) and Var(W) are respectively equal to

a) -16 and 44
b) -16 and 28
c) 24 and -44
d) None of these

$E(W) = 2 \times 2 - 4 \times 5 = -16$

$Var(W) = 4 \times 1 + 16 \times 2 + 2 \times (2) \times (-4) \times 0.5$
$= 28$

*** Suppose we are testing the claim μ > 70 at α 0.05. Type I error happens when we

a) We reject the claim given that it is true.
b) We reject the claim given that it is false.
c) Accept the claim given that the claim is true
d) Accept the claim given that the claim is false

X D